NUMBER

During the sensori-motor phase	***By the end of the sensori motor phase students typically:***	***As students move from sensori-motor phase to emergent phase***
Students develop awareness of people, objects and environments through sensory exploration (either independently or with support from an adult) They are given opportunity to control objects, people and their environment and to begin to make simple choices for activities to stop and start, for requesting events/objects. They have opportunity to respond to a wide range of stimuli, developing reliable responses and begin to communicate their likes/dislikes or ambivalence toward a stimuli. They use all their senses to experience and explore stimuli.	Recognise an obvious change happening very close to them and a start and stop in activity. Anticipate stimuli/events that occur over and over again. Reliably use appropriate movement to operate toys/switches Be able to shift attention between different objects/ actions Manipulate objects purposely Look for favourite objects when sees them hidden Open containers to find objects Use objects and materials according to their function Recognise when objects people are removed Turn takes in familiar situations Uses objects functionally Makes simple choices Solve simple problems where understanding the pattern is important	Have cause and effect and use this to solve increasingly complex problems Actively investigate a range of materials, environments and objects. Identify missing objects or parts of familiar objects, or when an item has been removed or hidden from a group of up to 3 objects. Requests 'more', through signing, gesture, symbols etc Finds 2 objects the same Pairs objects e.g. straw to cup, hat to head,
Common misconceptions and barriers to learning	***Supportive Strategies***	***Key Objectives***
Sensory impairment Communication skills Positioning and limited movement	Consistent repetition so learners generalise Consistent responses from adults Be skilled in observation of responses Supportive strategies that sensitively aid exploration Object /sensory cues for activities Limiting language Gradual extension and generalisation of tasks	**Functional use of an object** **Solve simple cause and effect problems** **Turn take** Footsteps recording **Mathsteps B20**

During the Emergent Phase	***By the end of the Emergent phase, students typically:***	***As students move from the Emergent phase to the Matching phase, they should demonstrate:***
Students reason about small amounts of physical materials, learning to distinguish small collections by size and recognising increases and decreases in them. They also learn to recognise and repeat the number words and to distinguish number symbols from other symbols. There is a growing recognition of what is the same and what is different between collections labelled with different numbers. As a result, students come to understand that number words and symbols can be used to signify the 'numerosity' of a collection.	• use 'bigger', 'smaller' and 'the same' to describe differences between small collections of like objects and between easily compared quantities. Make sets of equal quantities Matches objects to outlines. • anticipate whether an indicated change to a collection or quantity will make it bigger, smaller or leave it the same....if I take one away.? • distinguish spoken numbers from other spoken words • distinguish numerals from other written symbols • see at a glance how many are in small collections and attach correct number names to such collections (0-5) • connect the differences they see between collections of one, two and three with the number string: 'one, two, three.' • understand a request to share in a social sense and distribute items or portions.(Give every child a cup) Make piles of cubes Count movements and actions to 5 (5 claps, 4 steps etc) **These students recognise that numbers may be used to signify quantity.**	Quickly say how many objects are in a collection-up to 5 objects (subtilizing page 35) When counting small collections of objects they use the last number to answer 'how many?' (Counting Principles-trusting the count) Make a collection of objects or draw a set of objects e.g. draw 4 circles Get me 4 spoons. Count out a set of 5 and know that the last count is the answer to 'How many?' (Counting Principles-Trusting the count)
Common misconceptions	***Supportive strategies***	***Key objectives***
• may actually see at a glance how many there are in a small collection (e.g. *six pebbles*), yet may not be able to say the number names in order • may say a string of the number names in order (*one, two, three, four,* ...),but not connect them with actual objects • may be beginning to see how to use the number names to count, but may get the order of the names wrong may distribute items or portions in order to 'share', but may not be concerned about whether everyone gets some, the portions are equal, or the whole amount is used up • can tell by looking which of two small collections is bigger; but cannot say how much bigger. Students can become confused because the count may be one, two, three, but the teacher is pointing to only one object as they count, posting objects into a transparent container can help this as they see the objects accumulate as each count is added.	Regularly use or have on display number arrangements e.g. playing cards, dominoes, dice Help students see the familiar one-ness and two-ness in themselves e.g. 1 head, 1 nose, 1 mouth, 2 hands, 2 feet, 2 eyes, 2 ears etc Help students to see portioning at its early stages e.g. 3=2 + 1 by grouping people, objects etc Regularly find opportunities to rote count, count out actions/objects and label sets of objects. Give students opportunity to imagine/visualise numbers – I am thinking of an ice-cream, now there are 2 ice creams etc	**Subitise up to 5 objects** **Demonstrate cardinality with collections of up to 5** **Uses 1:1 correspondence when counting out objects to 5** **Recording-C22-H29 Mathsteps recording**

During the Matching Phase	***By the end of the Matching phase, students typically:***	***As students move from the Matching phase to the Quantifying phase, they should be able to demonstrate:***
Students use numbers as adjectives that describe actual quantities of physical materials. Through stories, games and everyday tasks, students use one-to-one relations to solve problems where they can directly carry out or imagine the actions suggested in the situation. They learn to fix small collections to make them match, 'deal out' collections or portions, and to respect most of the principles of counting. As a result, students learn what people expect them to do in response to requests such as: How many are there? Can you give me six forks? How many are left? Give out one (two) each. Share them.	• recall the sequence of number names at least into double digits • know how to count a collection, respecting most of the principles of counting • understand that it is the last number said which gives the count • understand that building two collections by matching one to one leads to collections of equal size, and can 'fix' one collection to make it match another in size • compare two collections one to one and use this to decide which is bigger and how much bigger • solve small number story problems which require them to add some, take some away, or combine two amounts by imagining or role playing the situation and counting the resulting quantity • share by dealing out an equal number of items or portions to each recipient, cycling around the group one at a time or handing out two or three at a time. Makes arrays or uses arrays to show simple multiplication and division e.g. pairs of shoes/socks to show 3 x 2 **These students use one-to-one relations to share and count out**	They match oral and written numbers names and can write recognisable versions of them at least to 12 and further Say the number names in order and respect order when counting objects. (Counting Principles p22) Distribute 8 sweets equally among 4 people. Give 1 more or 1 less than a given number up to 12 using recall Partition numbers to 10 Give a correct number of objects correctly on first request –up to 12 objects ('Get me' Task page 27) Distribute items fairly using a count to ensure enough resources e.g. count the children in the class and get enough cups. (Ice cream task p27) Counts a group or objects in ones (skip counting p29) Accurately and quickly identifies the bigger group of 2 collections (The More Game p34) Solve number addition and subtraction problems using stories(How many? Page 92) Solve multiplication and division problems using stories (Kangaroos and bananas-Kangaroos page only page 116/117) Use known number facts to add and subtract 1 or 2 objects. (Blocks in a Box page 102) Can find halves and 'not halves' Be able to identify and place objects using ordinal numbers first to third. Show me the second person in the line…Stand in third place.
Common misconceptions	***supportive strategies***	***Key Objectives***
They will often do not spontaneously use counting to compare two groups in response to questions, such as: Are there enough cups for all students? • may 'skip count' but do not realise it gives the same answer as counting by ones and, therefore, do not trust it as a strategy to find how many • often still think they could get a different answer if they started at a different place, so do not trust counting on or counting back • often can only solve addition and subtraction problems when there is a specific action or relationship suggested in the problem situation which they can directly represent or imagine	Give pupils opportunity to count of from a given number Practise with adding and subtracting zero Have experience of splitting amounts and developing number facts Ensure pupils see that addition can be done either way e.g. 6+4 is the same as 4+6 (commutative law)	**Count out at least 10 objects using 1:1 correspondence** **Order digits to 10** **Mathsteps-I20 -J23**

• have difficulty linking their ideas about addition and subtraction to situations involving the comparison of collections e.g. find the difference between… • may lay out groups to represent multiplicative situations, but do not use the groups to find out how many altogether, counting by ones instead (count all). • may represent division-type situations by sharing out or forming equal groups, but become confused about what to count to solve the problem, often choosing to count all the items • may deal out an equal number of items or portions in order to share, but do not use up the whole quantity or attend to equality of the size of portions • often do not realise that if they have shared a quantity, then counting one share will also tell them how many are in the other shares • may split things into two portions and call them halves but associate the word 'half' with the process of cutting or splitting and do not attend to equality of parts. When adding 2 sets ob objects pupils often cannot 'hold' the number of the previous count.		

During the Quantifying Phase	***By the end of the Quantifying phase, students typically:***	***As students move from the Quantifying phase to the Partitioning phase, they should be able to demonstrate the ability to:***
Students reason about numerical quantities and come to believe that if nothing is added to, or removed from, a collection or quantity, then the total amount must remain the same even if its arrangement or appearance is altered. As a result, students see that the significance of the number uttered at the end of the counting process is that it does not change with rearrangement of the collection or the counting strategy. They interpret small numbers as compositions of other numbers. Also as a result, they develop the idea that constructing fair shares requires splitting the whole into equal parts without changing the total quantity and so begin to see the part-whole relations that link sharing and fractions.	• without prompting, select counting as a strategy to solve problems such as: Are there enough cups? Who has more? Will it fit? • use materials or visualise to decompose small numbers into parts empirically; e.g. 8 is the same as 5 with 3 • find it obvious that when combining or joining collections counting on will give the same answer as starting at the beginning and counting the lot • make sense of the notion that there are basic facts; e.g. 4 + 5 is always 9 no matter how they work it out or in what arrangement • select either counting on or counting back for subtraction problems depending on which strategy best matches the situation • can think of addition and subtraction situations in terms of the whole and the two parts and which is missing • write number sentences that match how they think about the story line (semantic structure) for small number addition and subtraction problems Show 2 digit numbers using bundles of straws and single straws. • realise that repeated addition or skip counting will give the same result as counting by ones • realise that if they share a collection into a number of portions by dealing out or continuous halving and use up the whole quantity, then the portions must be equal regardless of how they look • understand that the more portions to be made from a quantity, the smaller the size of each portion. Use arrays to solve multiplication and division problems. **These students use part-part-whole relations for numerical quantities**	Continue the count beyond 20 continuing the 1-9 pattern within a decade e.g. 31,32,33,34,35… but may need help bridging across 39-40 Can count larger amounts by skip counting e.g. in groups (Skip counting p29) Partition numbers using real objects e.g. 9 is made up of 4 and 5, 3 and 6, 8 and 1 etc (Hide the jellybean page 37) (Emus, rabbits, sheep page 38) Add 2 digit and 1 digit numbers and add three single digit numbers up to 20 using concrete objects, counting on, number-lines etc (Number Tiles page 98) Solve number problems using knowledge of inverse calculations and partitioning (Kangaroos and bananas-comparing bananas page 118) (Change Task p136) Count back from 20 to 0 Have instant recall of number bonds to 10 Recognises odd and even numbers and can use 2 numerals to create an odd number or an even number. Use the language of: equal to, more than, less than (fewer), most, least
Common misconceptions	**Supportive strategies**	***Key Objectives***
• often cannot decompose into parts numbers that they cannot visualise or represent as quantities, so have difficulty in partitioning larger numbers to make calculation easier; e.g. students need to count forwards or backwards by ones to find the difference between 25 and 38	• although understanding that the two halves they have formed by dealing out or splitting must be equal, may think that a half formed one way could be bigger than a half formed another way • may ignore the size of portions when choosing fraction names; e.g. describing one part in seven as one seventh regardless of whether the seven portions are equal	**Know all bonds to 10** **Demonstrates conservation and cardinality of number** **Partitions numbers <10** **MATHSTEPS J24-k28**

can count equal groups by physically or mentally laying out each group, but think of and treat each group as distinct from the others • often believe that for halves there must be exactly two pieces; e.g. students may deny the equality of one half and two quarters unless the two quarters are 'stuck back together' Don't undertand the missing number in an equation so when faced with _+3+10, they add the 2 numbers they have and find the answer is 13. • often use strategies based on materials, counting on or counting back to solve addition and subtraction problems, but do not link these strategies or different problem types to a single operation (either + or –) • may be unable to use the inverse relationship between addition and subtraction to choose the more efficient of counting on or counting back for solving particular problems • often write their number sentences after they have solved the problem with materials, counting or basic facts, so they may be unable to write number sentences in advance when needed for problems involving larger numbers	• often do not link sharing to unit fractions and may think that eighths are bigger than thirds because 8 is bigger than 3. When adding three numbers together help children see which numbers they could start with more easily e.g. 6+9+4- you do not have to add them in order-using bonds to 10 the 6+4=10 is easy, and then we just have to add 10 and 9. Teach students to add near multiples of 10 e.g. 9 and 11 by adding 10 and then adding or subtracting 1.	

During the Partitioning Phase	***By the end of the Partitioning phase, students typically:***	***As students move from the Partitioning phase to the Factoring phase, they:***
Students come to see the significance of whole numbers having their own meaning independent of particular countable objects. They learn to use part-whole reasoning without needing to see or visualise physical collections. As a result, students see that numbers have magnitudes in relation to each other, can interpret any whole number as composed of two or more other numbers, and see the relationship between different types of addition and subtraction situations. Also as a result, students see that numbers can be used to count groups and that they can use one group as a representative of other equal groups. They trust, too, that appropriate partitioning of quantities must produce equal portions.	• can compare whole numbers using their knowledge of the patterns in the number sequence, and think of movements between numbers without actually or mentally representing the numbers as physical quantities • make sense of why any whole number can be rewritten as the addition of other numbers • partition at least two- and three-digit numbers into standard component parts (e.g. 326 = 300 + 20 + 6) without reference to actual quantities • count up and down in tens from starting numbers like 23 or 79 • write suitable number sentences for the range of addition and subtraction situations • use the inverse relationship between addition and subtraction to make a direct calculation possible; e.g. re-interpret 43 – 27 as 'what do you have to add to 27 to get 43' and so count on by tens and ones • can double count in multiplicative situations by representing one group (e.g. by holding up four fingers) and counting repetitions of that same group, simultaneously keeping track of the number of groups and the number in each group • find it obvious that two different-shaped halves from the same size whole must be the same size and are not tricked by perceptual features • use successive splits to show that one half is equivalent to 2 parts in 4, 4 parts in 8, etc. and expect that if the number of portions is doubled, they halve the size of each portion • partition a quantity into a number of equal portions to show unit fractions and, given a particular quantity, will say that one third is more than one quarter. **These students use additive thinking to deal with many-to-one relations.**	Read, write, and say numbers in order to 100 and beyond and count back from 100 (Up to and over 100 page 16) Partition 2-digit numbers into tens and ones and be able to give the value e.g. 26 is made up of 2 tens and 6 units or 20 and 6. (Dinosaurs page 46) (Sweets page 49) Count in steps of 2's, 5;s and 10's from 0 Mentally add and subtract one-digit and two-digit numbers to 20, including 0 Have instant recall of 1 more and 1 less to 100 Know number bonds to 20 and halves and doubles to 20 Estimate the size of a collection up to 30 accurately. Count on or up in tens from any given number e.g. 37, 47, 57… Partition any 2-digit number in to other numbers e.g. 43 can be 40+3 or 30+13 or 20+23 etc Round numbers up or down to the nearest 10 or 100 Use a decimal point for money notation Find ½ and ¼ of a group of objects or a quantity Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 17 = ? − 29
Common misconceptions	***Supportive Strategies***	***Key Objectives (y1)***
• can 'work out' a non-standard partition (e.g. 47 = 30 + 17), but they may not see it as following automatically from the way numbers are written	Use place value cards, charts and base 10, bundles of straws etc to enable pupils to clearly see 23 = 2 sets of 10 and 3 ones Teach doubling and halving together and demonstrate that they are opposite calculations	**Count, read and write numbers to 100.** **Recognise, find and name a ½ and ¼** **Partition 2 digit numbers in to tens and ones** **Instant recall of bonds to 20** **Instant recall of doubles and halves to 20**

• often do not realise that the digit in the tens (hundreds) place refers to groups of ten (hundred) even when they correctly use the labels 'ones', 'tens' and 'hundreds' • often write related divisions and multiplications (e.g. 6 x 3 = 18, 18 ÷ 3 = 6, 18 ÷ 6 = 3) by working each out, are unable to use the inverse relationship between division and multiplication to work out an unknown quantity • may not understand why grouping can be used to solve a sharing problem • can write multiplication number sentences for problems which they can think of as 'lots of', but may only solve other types of multiplicative problems with materials or by counting • do not understand why multiplication is commutative; e.g. they often do not see that four piles of 13 must be the same amount as 13 groups of 4	Rehearse number facts to 20 regularly.	**Mathsteps L20-M27**

During the Factoring Phase – Pathway 3	***By the end of the Factoring phase, students typically:***	***As students move from the Factoring phase to the Operating phase, they:***
Students extend their additive ideas about whole numbers to include the coordination of two factors needed for multiplicative thinking. They learn to construct and coordinate groups of equal size, numbers of groups and a total amount. Students also learn to visualise multiplicative situations in terms of a quantity arranged in rows and columns (an array). As a result, students see the significance of the connection between groups of ten or groups of one hundred and the way we write whole numbers. They are able to relate different types of multiplication and division situations involving whole numbers. They also link the ideas of repeating equal groups, splitting a quantity into equal parts and fractions.	• use their knowledge that, for example, the 2 being in the tens place in 426 tells us that it refers to two groups of ten, to generate alternative partitions • sustain a correct whole number place-value interpretation in the face of conflicting information • are flexible in their mental partitioning of whole numbers, confident that the quantity has not changed • understand that a number can be decomposed and re-composed into its factors in a number of ways without changing the total quantity • find it obvious that if 3 rows of 5 is 15, then both 15 divided by 3 and one third of 15 are 5 • can visualise an array to see, for example, that five blue counters is one third of a bag of 15 counters, both because 15 can be split into three parts each of five and one in every three counters will be blue • visualise or draw their own diagrams to compare fractions with the same denominator (e.g. 37 and 57) or simple equivalences (e.g. 14 and 38) • relate fractions and division knowing, for example, that 3 4 can be thought of as 3 ÷ 4 and 3 things shared among 4 students has to be 3/4 • know that they can choose between multiplication or division to make calculating easier • understand why grouping and sharing problems can be solved by the same division process • interpret multiplication situations as 'times as much' and so can see that 12 is three times as much as 4, and 8 is ten times smaller than 80. • select an appropriate multiplication or division operation on whole numbers including for problems that are not easily interpreted as 'lots of'; e.g. combination and comparison problems Pupils are introduced to the multiplication tables. • can see why multiplication of whole numbers is commutative; e.g. know without calculating, that four piles of nine objects must be the same amount as nine piles of four objects. **These students think both additively and multiplicatively about numerical quantities.**	Order and compare numbers to 100 Read, write and say 3 digit numbers including those with zero as a place holder e.g. 306 (Flexible numbers page 62) Partition each number into hundreds, tens and units. Have instant recall of all addition and subtraction facts to 20 and derive number facts for 100 e.g. if 10-1=9 then 100-10=90 Mentally add 10 to any 2-digit number . Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even number patterns Use mental calculations to solve problems explaining their working (Find the solutions page 106) (How did you do it? set A page 32) Begin to use x and division and = symbols to layout calculations. **Begin to make some formal written calculations** for + and – using columns to show place value. Find the factors of a given number e.g. 12 = 3x4, 2x6, 1x12 Compare 2 numbers to find the difference (How much taller? Page 123) Use the inverse relationship between addition and subtraction to solve problems (Empty Boxes page 129/130) (Change task page 137) Solve multiplication and division problems (Story problems page 146 and 147) (How many? 3 and 4 page 95/96) Convert decimals to fractions (tenths and hundredths and also common fractions ½, ¼ and ¾) Recognise, find, name and write fractions e.g. 1/3, ¼, 2/4 and ¾ of a length, shape or set of objects. Recognise equivalence of 2/4 and ½. Understand the multiplicative nature of place value for whole numbers when using a PV grid Use and order negative numbers Derive addition and subtraction bonds to 100 Count in3's and 4's

Common misconceptions	Supportive strategies	Key Objective Y2
• often continue to rely largely on their knowledge of the 'named' places in reading and writing numbers, so have difficulty writing numbers with more than four digits • often are unable to select a common partitioning (denominator) to enable two fractions to be compared or combined unless an equivalence they already know is involved • may be unable to select an appropriate operation in situations where they cannot think of the multiplier or divisor as a whole number • may resist selecting division where the required division involves dividing a number by a bigger number • often believe that multiplication 'makes bigger' and division 'makes smaller'. Confusion in place value can be seen when students make errors in ordering similar looking numbers 212, 221, 122 Confusion with place value can also be seen when pupils show errors with zero, sometimes pupils feel it doesn't matter and can be ignored.	Pupils need to practise addition and subtraction to 20 so that they become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 − 7 = 3 and 7 = 10 − 3 to calculate 30 + 70 = 100; 100 − 70 = 30 and 70 = 100 − 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This helps them establish commutativity and associativity of addition. Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).	**Compare numbers to 100 using <,> and =** **Have instant recall of all addition and subtraction facts to 20 and derive facts for 100** **Partition 3 digit numbers in to H,T and U** **Add and subtract mentally with one and two digit numbers.** **Use inverse relationship between addition and subtraction.** **Know the 2x, 5x and 10x tables by heart.** **Recognise and name fractions 1/3, ¼,1/2 and ¾ and find fractions of shape, length or quantity.**

During the pre-Operating Phase- Pathway 4	*By the end of the pre-Operating phase, students typically:*	*As students move from the pre-operating phase to the Operating phase, they:*
Students begin to understand place value for whole numbers and decimals. They begin to see that whole units can be divided into smaller parts. They begin to understand place value and link to x10, x100, x1000 etc. They rote learn multiplication facts but have a sound understanding of how multiplication works. They can explain fractions visually showing equal fractions	• Use the decimal point to record money notation and measure • generalise their understanding of whole number place value to include the cyclical pattern beyond the thousands, so can they begin to read, write and say common fractions and decimals • Use their knowledge of place value to x and divide by moving digits to the left or right • realises that when multiplying by 1 the number remains the same and when multiplying by 0 the answer is always zero. • select an appropriate number of partitions to enable a quantity; e.g. 15, to be shared into two different numbers of portions; e.g. either 5 or 3 • produce their own diagrams to compare or combine two fractions, ensuring that both fractions (e.g. 2 /3 and 1/4) are represented on identical wholes • split and recombine fractions visually or mentally to add or subtract; e.g. 1/2 + 1/4 is (1/4 + 1/4) + 1/4 = 3/4 • can write suitable number sentences for the full range of multiplication and division situations involving whole numbers • Know multiplication tables for 3,4 and 8 Move from mental and informal written methods of addition and subtraction to formal column calculations. **These students can think of multiplications and divisions in terms of operators.**	Read and write numbers up to 1,000 in numerals and in words solve number problems and practical problems involving these ideas. (Read, write and say whole numbers page 42) Partition 4-digit numbers into thousands, hundreds, tens and units. Demonstrate an understanding that numbers can be read in different ways e.g. 1100 can be read as one thousand, one hundred or eleven hundred. (Flexible numbers page 62) Mentally add 1,10,100 or 1000 to a given number. Count from 0 in multiples of 4, 8, 50 and 100; Find 10 or 100 more or less than a given number Compare and order numbers up to 1,000 Multiply and divide given whole numbers by 1,10 and 100 (800 game page 55) Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction Estimate the answer to a calculation and use inverse operations to check answers Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction (Find the solutions page 106-108) Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and **progressing to formal written methods** (How did you do it? Page 32 set B) Solve problems, including missing number problems,(Empty Boxes page 130) involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators Recognise and show, using diagrams, equivalent fractions with small denominators

		Add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$] Compare and order unit fractions, and fractions with the same denominators Read, write and order negative numbers
Common misconceptions	***Supportive strategies***	Key Objectives Y3
Pupils often ignore the need to draw two fractions on identical wholes in order to compare or combine them • have developed ideas about decimals based on daily use for money and measures, so may think the decimal point separates two whole numbers, where the whole numbers refer to different-sized units; e.g. when referring to money, they may read 6.125 as if the 6 is pounds and the 125 is pence and thus 'round' it to £7.25 or say that 6.125 > 6.25 • may rightly think of decimals as another way to represent fractional numbers but, for example, think 0.6 is one sixth Pupils may have difficulty in understanding that 0 can be larger than -1. may believe that to show a fraction of a collection the denominator must match the total number of items and will be unable, for example, to recognise six parts in 18 as one third • may think of 1/3 only as one part out of a collection or quantity, which has been split into three equal parts, but do not also recognise it as one in each three • may think of fractions as quantities rather than numbers and not see the significance of using the same unit as the basis for comparing fractions, so do not see why 1/3 must be bigger than 1/4 • may see fractions, such as three quarters, literally as three pieces each of one quarter and will not accept one piece which is three quarters of the whole.	Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables. Never move the decimal point to multiply or divide-this cannot be moved Never talk to pupils about adding a zero to multiply by 10 as this does not apply when multiplying decimals. Using a variety of representations, including those related to measure, pupils continue to count in 1s, 10s and 100s, so that they become fluent in the order and place value of numbers to 1,000. This will help pupils with conversions in units of measure. Pupils connect tenths to place value, decimal measures and to division by 10. Show this practically-e.g. dividing a metre rule in to 10 equal parts, using base 10 etc This will help provide concrete examples.	**Compare and order numbers to 1000** **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** **Add and subtract numbers mentally** **Round numbers to nearest hundred, tens or units** **Add and subtract using formal written methods** **Know the 3,4, and 8 x tables** **Count up and down in tenths and understand that tenths are divisions of an object or quantity in 10 = parts** **Compare and order simple fractions.** **Find fractions of a set of objects.** **Add and subtract fractions less than 1 with common denominators.**

During the Operating Phase	***By the end of the Operating phase, students typically:***	***As students move from the pre-operating phase to the Operating phase, they:***
Students learn to interpret multipliers as 'times as much as' or 'of' rather than simply counters of groups, so can think of them as 'operators' that need not be whole numbers. Students also come to see that any number can be thought of as a unit which can be repeated or split up any number of times. As a result, students see how the intervals between whole numbers can be split and re-split into increasingly smaller intervals and realise the significance of the relationship between successive places. For example, the value of each place is ten times the value of the place to its right and one tenth the value of the place to its left. Also as a result, students learn to make multiplicative comparisons between numbers, deal with proportional situations, and integrate their ideas about common and decimal fractions.	• represent common and decimal fractions both smaller and greater than 1 on a number line • generalise their understanding of whole number place value to include the cyclical pattern beyond the thousands, so can read, write and say any whole numbers • use their understandings of the relationship between successive places to order decimal numbers regardless of the number of places • use the cyclical pattern in the places to count forwards and backwards in tenths, hundredths, thousandths, including up and over whole numbers • are flexible in partitioning decimal numbers • realise that for multipliers smaller than 1, multiplication makes smaller, and for divisors smaller than 1, division makes bigger • construct successive partitions to model multiplication situations; e.g. *I took half the cake home and then ate one third of it* • recognise the need to multiply in situations where the multiplier is a fractional number • can write suitable number sentences for the full range of multiplication and division situations involving whole numbers, decimals and fractions. **These students can think of multiplications and divisions in terms of operators.**	Read, write and say whole numbers into tens and hundreds of thousands(Read, write and say whole numbers page 42) Place fractions and decimals on a number line. Rewrite decimals as fractions(1/2,1/4,3/4,tenths and hundredths) Compare and order the above fractions and decimals.(Circle the biggest page 58) Understand the multiplicative nature of place value for decimals Count from 0 in multiples of 4, 8, 50 and 100; Find 10 or 100 more or less than a given number Recognise the place value of each digit in a 5-digit number (100s, 10s, 1s) Compare and order numbers up to 100,000 Identify, represent and estimate numbers using different representations Read and write numbers up to 100,000 in numerals and in words solve number problems and practical problems involving these ideas Use decimals practically to problem solve, understanding that rounding may be required for a sensible answer (Apples and money page 70) Use formal written methods for short multiplication Order and compare decimals (Library books page 72) Demonstrate an understanding that 0.2 can be read as 2/10, 20/100 or 200/1000 (Digit values page 74) Pupils understand the role of 0 in decimal numbers (Decimal numbers Page 92)
Common misconceptions and supportive strategies	***Supportive strategies***	***Key Objectives Y4***
Students may label the places to the right of the decimal point as tenths and hundredths and write 2.45 as 2 + 4/10 + 5/100 , for example, but cannot link this with other ways of writing the decimal, such as: 2 + 45/100 Students may think decimals with two places are always hundredths and write 2.45 as 2 + 45/100 , but do not link this with the pattern in whole-number place value and so do not see 2.45 as 2 + 4/10 + 5/100		**Count backwards through zero including negative numbers.** **Recognise place value in 4 digit numbers** **Round any number to the nearest 10, 100 or 1000** **Know all tables to 12x12** **Use number facts and place value to carry out mental calculations** **Use factor pairs** **Use formal written methods for short multiplication** **Use hundredths** **Find decimal equivalents for common fractions** **Divide 1 or 2 digit numbers by 10 and 100 using tenths and hundredths** **Round decimals with one decimal place to the nearest whole number.** **Compare numbers to 2 decimal places**

During the Operating Phase	***By the end of the Operating phase, students typically:***	***As students move from the pre-operating phase to the Operating phase, they:***
Students learn to interpret multipliers as 'times as much as' or 'of' rather than simply counters of groups, so can think of them as 'operators' that need not be whole numbers. Students also come to see that any number can be thought of as a unit which can be repeated or split up any number of times. As a result, students see how the intervals between whole numbers can be split and re-split into increasingly smaller intervals and realise the significance of the relationship between successive places. For example, the value of each place is ten times the value of the place to its right and one tenth the value of the place to its left. Also as a result, students learn to make multiplicative comparisons between numbers, deal with proportional situations, and integrate their ideas about common and decimal fractions.	• represent common and decimal fractions both smaller and greater than 1 on a number line • generalise their understanding of whole number place value to include the cyclical pattern beyond the thousands, so can read, write and say any whole numbers • use their understandings of the relationship between successive places to order decimal numbers regardless of the number of places • use the cyclical pattern in the places to count forwards and backwards in tenths, hundredths, thousandths, including up and over whole numbers • are flexible in partitioning decimal numbers • realise that for multipliers smaller than 1, multiplication makes smaller, and for divisors smaller than 1, division makes bigger • construct successive partitions to model multiplication situations; e.g. *I took half the cake home and then ate one third of it* • recognise the need to multiply in situations where the multiplier is a fractional number • can write suitable number sentences for the full range of multiplication and division situations involving whole numbers, decimals and fractions. **These students can think of multiplications and divisions in terms of operators.**	Read, write and say whole numbers into millions (Read, write and say whole numbers page 42) Place fractions and decimals on a number line. Rewrite decimals as fractions Compare and order fractions and decimals.(Circle the biggest page 58) Understand the multiplicative nature of place value for decimals and order decimals to 3 places Count from 0 in multiples of 4, 8, 50 and 100; Find 100, 1000 or 10,000 more or less than a given number Recognise the place value of each digit in a 6-digit number Compare and order numbers up to 100,000 Identify, represent and estimate numbers using different representations Read and write numbers up to 100,000 in numerals and in words solve number problems and practical problems involving these larger amounts Use decimals practically to problem solve, understanding that rounding may be required for a sensible answer (Apples and money page 70) Use formal written methods for short multiplication Order and compare decimals (Library books page 72) Demonstrate an understanding that 0.2 can be read as 2/10, 20/100 or 200/1000 (Digit values page 74) Pupils understand the role of 0 in decimal numbers (Decimal numbers Page 92) Use negative numbers in real contexts Find factors of numbers and use the vocabulary of factor, multiple, square, prime, cube.

Common misconceptions and supportive strategies	*Supportive strategies*	*Key Objectives Y5*
Students may label the places to the right of the decimal point as tenths and hundredths and write 2.45 as 2 + 4/10 + 5/100 , for example, but cannot link this with other ways of writing the decimal, such as: 2 + 45/100 Students may think decimals with two places are always hundredths and write 2.45 as 2 + 45/100 , but do not link this with the pattern in whole-number place value and so do not see 2.45 as 2 + 4/10 + 5/100		**Interpret negative numbers in context** **Recognise and use square and cube numbers, and know the notation** **Identify multiples and factors, including finding factor pairs and common factors** **Know prime numbers up to 19** **Multiply and divide numbers by 10, 100 or 1000, including decimals** **Use long multiplication for multiplying numbers of up to 4 digits by one or two digits** **Divide numbers using standard written short division** **Convert between mixed numbers and improper fractions** **Compare and order fractions whose denominators are multiples of the same number** **Identify, name and write equivalent fractions including tenths and hundredths** **Add and subtract fractions with denominators that are multiples of the same number** **Multiply proper fractions and mixed numbers by whole numbers with support** **Read, write, order, round and compare numbers with up to 3 decimal places** **Recognise % symbol and explain as a fraction with denominator 100 (parts out of 100)**
		Key Objectives Year 6
		Use negative numbers to calculate intervals across zero **Divide numbers using long division, interpreting remainders** **Use order of operations to carry out calculations** **Use common factors to simplify fractions** **Compare and order fractions** **Add and subtract fractions with different denominators and mixed fractions** **Multiply simple pairs of proper fractions** **Divide proper fractions by whole numbers** **Calculate decimal fraction equivalents for simple fractions** **Multiply a number with up to 2 decimal places by whole numbers** **Use written division with answers of up to 2 decimal places** **Solve problems with calculations of %**

		Recall and use equivalence between fractions, decimals and percentages **Solve problems using ratio using multiplication and division facts** **Use simple formulae** **Express missing number problems algebraically**